Weighted Voting Models for the HathiTrust Constitutional Convention

by Julia Lovett

I. Introduction and Scope

In March 2011, the HathiTrust Digital Library partner institutions will meet for a Constitutional Convention (CC). At this meeting, the partners will either develop a new governance model for HathiTrust or articulate a set of questions to frame a post-Convention discussion of a new governance model. Questions might include: What should the governing boards be, and how should they be constituted? Which institutions should have representative power, and how many representatives should there be per institution? Should HathiTrust have a new host institution and, if so, what criteria should be used to select a new host institution? These decisions will have a large impact on the future of HathiTrust, and should be arrived at through a method that accurately reflects varying levels of institutional investment in the project. The best way to ensure fair decision-making in a situation where voters are inherently unequal is to establish a weighted voting system (see Barrett and Newcombe, 1968). To this purpose, this paper explores the following questions:

- How is voting power calculated in a weighted voting system?
- What factors and principles should inform allocation of voting power?
- How do these models apply to HathiTrust?

The first question is the most straightforward one to answer, drawing from scholarly literature about *a priori* voting power, a subset of general voting theory and social choice theory. Voting power theory can be applied to "any collective body that makes yes-or-no decisions by vote." (Felsenthal and Machover, 1998) The classification *a priori* refers to voting power that is "determined without taking into consideration voters' prior bias regarding the bill voted upon, or the degree of affinity (for example, ideological proximity) between voters." (Felsenthal and Machover 1998) In other words, the systems that theorists have used to determine voting power depend on the assumption that every possible combination of votes in a committee is equally likely to occur. In reality, of course, not all combinations are equally likely. Certain scholars strongly object to the fact that *a priori* methods exclude external factors such as voters' past biases and affiliations (see Gelman et al., 2004; Margolis, 2004; Albert, 2003). This objection does not seem to present a problem for HathiTrust. Such biases and affiliations, if they exist, would be impossible to quantify without the existence of any voting prior to the CC.

Voting power theory's restriction to binary (two option) decisions is necessary for a number of reasons. First and foremost, a method of calculating weighted voting power for a system involving many options or complicated voting procedures has not been widely studied. Many voting theorists have detailed procedures with more than two choices and perhaps more than one result (Black, 1998; Dummett, 1984; Tideman, 2006; Brams, 1983), but these studies usually assume that constituents have equal votes. In addition, such studies often focus on elections and large-scale voting processes, as opposed to voting in committees. Felsenthal and Machover (1998) explicitly note these trends in the literature, describing the measurement of voting power as “orthogonal to the concerns of the general theory of voting.” (5) The most important aspect of a voting model for the HathiTrust CC is that each partner institution has a level of influence commensurate with its degree of investment in HathiTrust. Given existing methods of calculating voting power, the surest results can be drawn from binary voting. In addition, voting theory often assumes that a *simple majority* (more than half) will be required for a decision to pass. It is possible, of course, for the *quota*—the number of votes required to pass a decision—to be raised higher than a simple majority. Several studies have been made of the effect of various quotas on real voting bodies (Leech, 2003; Dreyer and Schotter, 1980). For the sake of simplicity, this paper will discuss voting theory based on a simple majority.

The more difficult questions to answer are the second and third in the above list: What factors and principles should inform allocation of voting power? How can these models be applied to HathiTrust? Whereas methods of calculating voting power have been widely cited and accepted by many theorists, there is no straightforward method detailing how to fairly allocate such power. The weighting factors seem to depend on the organization, and could include principles of fairness, monetary contributions, and a desire to balance larger and smaller powers. Case studies of voting in real organizations such as the United Nations, International Monetary Fund, and European Union Council of Ministers, could provide suggestions for HathiTrust.

II. Voting Power Theory

There are two widely accepted and cited methods for calculating *a priori* voting power. One was outlined by Penrose (1946) and later arrived at independently by Banzhaf (1965). The other method was first proposed by Shapley and Shubik (1954). Furthermore, Leech (2003) has demonstrated how to use these methods to work backwards and calculate voting weight based on predetermined voting power, according to the Banzhaf method. There are also some minor methods that have not been widely adopted and will not be discussed in this paper (see Felsenthal and Machover, 2004).

The history of voting power theory has been characterized by fits and starts, as many scholars have undertaken the problem of measuring *a priori* voting power while apparently unaware of prior work in the subject. Banzhaf independently arrived at the same system as Penrose, in ignorance of the latter’s 1946 paper. Coleman (1971) approached the problem unaware of both Shapley-Shubik and Banzhaf. (Felsenthal and Machover, 1998) Only relatively recently has the field come together in a coherent way.

In both the United States and Europe, interest in voting power theory has surged with changes in decision rules of large voting bodies. For example, in the European Union, many theorists argued for or against the use of *a priori* power measures as a guide for the re-weighting of votes in the Council of Ministers in 2001 (Pajala and Widgren, 2004). In the United States, the topic of voting power enjoyed the greatest popularity in the 1960’s when many scholars advocated applying weighted voting to state legislatures. (Felsenthal and Machover, 2004; Leech, 2003) It was during this controversy that John Banzhaf first argued against equating voting weight with voting power.

A. Penrose and Banzhaf

In his seminal article "Weighted Voting Doesn't Work: A Mathematical Analysis," John Banzhaf (1965) argues that "voting power is not proportional to the number of votes a legislator may cast." (318) In other words, a greater voting weight does not automatically translate into greater power. Banzhaf crafted this argument in response to widespread suggestions at the time that weighted voting, in which legislative representatives would hold voting weights proportional to the population of their respective districts, could be implemented as an alternative to reapportioning the legislative seats. The resulting distribution of power, proponents argued, would be the same in either case.

Banzhaf demonstrates the fallacy of equating weight with power through a series of examples and mathematical reasoning. He asserts, "it would be more effective to think of voting power as the ability of a legislator, by his vote, to affect the passage or defeat of a measure." (318) A legislator's power lies in the chance that his individual vote will swing the collective vote. Those chances are indeed highly dependent on the relative voting weights in the committee, but not in a predictable or intuitive way. Banzhaf presents the following table (p. 339) as one real example where seemingly appropriate voting weights result in radically inappropriate power distribution:

NASSAU COUNTY SYSTEM OF WEIGHTED VOTING 1964 [52]

Municipality	Population (1960)	Number of Weighted Votes	Number of Combinations in Which Each Legislator Can Affect The Outcome
Hempstead (No. 1)	728,625	31	16
Hempstead (No. 2)		31	16
North Hempstead	213,225	21	00
Oyster Bay	285,545	28	16
Glen Cove	22,752	2	00
Long Beach	25,654	2	00

In order to tally the numbers in the last column, Banzhaf has taken all the possible combination of votes and looked at the combinations where the legislator could swing the vote. He assumes that a *simple majority* (more than half) will cause the bill to pass. The most surprising result is that North Hempstead, with a population of 213,225 and 21 weighted votes, can never be the pivotal vote and therefore effectively has no voting power.

Banzhaf's argument echoes the work of L. S. Penrose (1946), whose article "The Elementary Statistics of Majority Voting" first undertook a serious study of *a priori* voting power. Penrose writes, "the power of the individual vote can be measured by the amount by which his chance of being on the winning side exceeds one half." (53) Paraphrased by Felsenthal and Machover (2004), Penrose's basic argument (the same as Banzhaf's) is that "the more powerful a voter is, the more often will the outcome go the way s/he votes." (3) Banzhaf, who was apparently unaware of Penrose's work, explains this same concept with a simple mathematical argument:

> More explicitly, in a case where there are *N* legislators, each acting independently and each capable of influencing the outcome only by means of his votes, the ratio of the power of legislator X to the power of legislator Y is the same as the ratio of the number of possible voting combinations of the entire legislature in which X can alter the outcome by changing his vote to the number of combinations in which Y can alter the outcome by changing his vote. (331)

While Penrose framed his argument around the absolute probability of a voter's success, Banzhaf was more interested in representing relative power. The following table (p. 342) shows his method of listing all possible voting combinations and tallying "combinations in which each legislator casts a decisive vote:"

Table A. Analysis of Voting Power in Example 3

	Exhaustive List of All Possible Voting Combinations					Winning Combinations	Combinations in Which Each Legislator Casts a Decisive Vote				
	H	I	J	K	L		H	I	J	K	L
	5	5	3	3	1		5	5	3	3	1
1	Y	Y	Y	Y	Y	√					
2	Y	Y	Y	Y	N	√					
3	Y	Y	Y	N	Y	√					
4	Y	Y	Y	N	N	√	√	√			
5	Y	Y	N	Y	Y	√					
6	Y	Y	N	Y	N	√	√	√			
7	Y	Y	N	N	Y	√	√	√			
8	Y	Y	N	N	N	√	√	√			
9	Y	N	Y	Y	Y	√	√				
10	Y	N	Y	Y	N	√	√		√	√	
11	Y	N	Y	N	Y	√	√		√		√
12	Y	N	Y	N	N			√		√	√
13	Y	N	N	Y	Y	√	√			√	√
14	Y	N	N	Y	N			√	√		√
15	Y	N	N	N	Y			√	√	√	
16	Y	N	N	N	N			√			
17	N	Y	Y	Y	Y	√		√			
18	N	Y	Y	Y	N	√		√	√	√	
19	N	Y	Y	N	Y	√		√	√		√
20	N	Y	Y	N	N		√			√	√
21	N	Y	N	Y	Y	√		√		√	√
22	N	Y	N	Y	N		√		√		√
23	N	Y	N	N	Y		√		√	√	
24	N	Y	N	N	N		√				
25	N	N	Y	Y	Y		√	√			
26	N	N	Y	Y	N		√	√			
27	N	N	Y	N	Y		√	√			
28	N	N	Y	N	N						
29	N	N	N	Y	Y		√	√			
30	N	N	N	Y	N						
31	N	N	N	N	Y						
32	N	N	N	N	N						
						Totals	16	16	8	8	8
							H	I	J	K	L

Y=Yea vote.
N=Nay vote.

Again, there are some surprising results: Voter L, with a weight of one, has just as much real voting power as J and K, who each have a weight of three. In order to determine the relative voting power of H, I, J, K, and L, one would divide an individual's voting power by the total amount of voting power in the assembly. Essentially, this calculation results in the individual voter's share of the total available voting power. Converting the numbers to percentages, so that the entire weight adds up to 1, results in what is termed the Banzhaf Index (Felsenthal and Machover, 2004, p. 5). Felsenthal and Machover (2004, p. 6) demonstrate Banzhaf Index values for the European Union Council of Ministers in the following table:

Table 3: The Banzhaf Index (β) under QMV in the EU Council of Ministers, First Five Periods

Country	1958	1973	1981	1986	1995
Germany	0.238	0.167	0.158	0.129	0.112
Italy	0.238	0.167	0.158	0.129	0.112
France	0.238	0.167	0.158	0.129	0.112
Netherlands	0.143	0.091	0.082	0.067	0.059
Belgium	0.143	0.091	0.082	0.067	0.059
Luxembourg	0.000	0.016	0.041	0.018	0.023
UK		0.167	0.158	0.129	0.112
Denmark		0.066	0.041	0.046	0.036
Ireland		0.066	0.041	0.046	0.036
Greece			0.082	0.067	0.059
Spain				0.109	0.092
Portugal				0.067	0.059
Sweden					0.048
Austria					0.048
Finland					0.036
Total	1.000	1.000	1.000	1.000	1.000

If the HathiTrust partners accept the Banzhaf Index as a measure of relative voting power, such an index would be simple to compute for any given voting weights. The more difficult part would be working backwards to find voting weight, given a predetermined power distribution. Leech (2003) points out, "There have been many studies of the distribution of a priori power in actual voting bodies where the decision rule and allocation of votes to voting members is given but relatively few where the approach has been used as a tool for designing weighted voting systems." In this same paper, Leech demonstrates how to use the Banzhaf index to design a voting system. These calculations are possible with some mathematical work, discussed in more detail in the section below, "Determining Weight for a Given Banzhaf Power Allocation."

B. Shapley and Shubik

The Shapley-Shubik (1954) method is the other widely accepted system of determining voting power. At first glance, this method seems to replicate Banzhaf's approach. Shapley and Shubik state, "Our definition of the power of an individual member depends on the chance he has of being critical to the success of a winning coalition." (787) This "chance of being critical" is important in Banzhaf's method as well. But their concept of voting power begins to sound a bit different from Banzhaf's: "It is possible to buy votes in most corporations by purchasing common stock. If their policies are entirely controlled by simple majority votes, then there is no more power to be gained after one share more than 50% has been acquired." (788) Whereas Penrose and Banzhaf both described voter power as the power to influence a decision, Shapley-Shubik describe voter power as a share in an expected payoff. There is a fundamental conceptual difference between the two methods; whereas the Banzhaf method is based on probability, the Shapley-Shubik method is based on game theory.

Felsenthal and Machover (1998, 2004) strongly argue this point about the conceptual differences between the two methods. Shapley and Shubik's view of power, they argue, "is that passage or defeat of the bill is merely the ostensible and proximate outcome of a division. The real and ultimate outcome is the distribution of a fixed purse (the prize of power) among the victors in the case where a bill is passed." (10) Voters according to Penrose and Banzhaf are policy-seeking; voters according to Shapley and Shubik are office-seeking. (Felsenthal and Machover 2004, p. 11) The idea of voter motivation is something that comes up often in the general theory of voting (see Coleman, 1971; Dummett, 1984). This paper will not explicitly detail those arguments, but it helps to be aware that certain ideas about voter motivation underlie both the Penrose-Banzhaf and Shapley-Shubik methods. In deciding on a method of calculating voting power in the HathiTrust CC, the partners should consider which notion of voter motivation fits with HathiTrust. Most likely, the partners will all be interested in what is best for HathiTrust, and therefore could be regarded as policy-seeking rather than office-seeking.

Beyond having conceptual differences, the Banzhaf and Shapley-Shubik methods sometimes produce different numerical results when applied to the same set of voter weights. The differences can be seen in this table from Lane and Berg (1999), detailing voting powers in Germany (p. 314) according to the Penrose measure (BI), Banzhaf Index (Banzhaf norm.), and Shapley-Shubik Index (SSI):

Table 1. Constitutional Change in Germany: Votes and Voting Power of the States

Members	First Proposal				Second Proposal			
	Votes	BI	Banzhaf norm.	SSI	Votes	BI	Banzhaf norm.	SSI
Nordrhein-Westfalen	8	0.318	0.104	0.105	6	0.272	0.067	0.067
Bayern	7	0.274	0.090	0.090	6	0.272	0.067	0.067
Baden-Württemberg	7	0.274	0.090	0.090	6	0.272	0.067	0.067
Niedersachsen	7	0.274	0.090	0.090	6	0.272	0.067	0.067
Hessen	6	0.237	0.078	0.078	4	0.183	0.059	0.059
Rheinland-Pfalz	5	0.191	0.062	0.062	4	0.183	0.059	0.059
Berlin	5	0.191	0.062	0.062	4	0.183	0.059	0.059
Schleswig-Holstein	4	0.152	0.050	0.050	4	0.183	0.059	0.059
Hamburg	3	0.114	0.037	0.037	3	0.129	0.042	0.041
Saarland	3	0.114	0.037	0.037	3	0.129	0.042	0.041
Bremen	3	0.114	0.037	0.037	3	0.129	0.042	0.041
Mecklenburg-Vorpo	4	0.152	0.050	0.050	4	0.183	0.059	0.058
Brandenburg	4	0.152	0.050	0.050	4	0.183	0.059	0.058
Sachsen-Anhalt	4	0.152	0.050	0.050	4	0.183	0.059	0.058
Thüringen	4	0.152	0.050	0.050	4	0.183	0.059	0.058
Sachsen	5	0.191	0.062	0.062	4	0.183	0.059	0.058

One potential drawback of using the Shapley-Shubik Index is that it is much more difficult to calculate than the Banzhaf Index. In Shapley and Shubik's original article (1954), after giving an example of an individual voter's power in a certain situation, they state: "The calculation of this value and the following [values] is quite complicated, and we shall not give it here." (789) Perhaps the simplest explanation is given by Dixon (1983, p. 298), who writes:

> This measure reflects the prior probability that an individual will cast the deciding vote on any issue by being the last member to join a minimal winning coalition . . . To determine the expectation that an individual will pivot, one must consider all of the possible sequences (n! in an n member voting body) in which a minimal winning coalition might form. For any member i, the Shapley-Shubik power index, ϕ, may be defined as
>
> $$\phi = (\text{total pivots for } i) / n!$$

The Shapley-Shubik Index takes into account the "possible sequences," the order in which votes occur. Again, this points to a conceptual difference as well as a methodological difference between the Shapley-Shubik method and the Banzhaf method: Whereas Banzhaf focuses on combinations of votes, the Shapley-Shubik method focuses on possible sequences of votes. The pivotal member, according to Shapley-Shubik, is the last member whose vote forms a "minimal winning coalition," rendering the subsequent votes meaningless. Again, for HathiTrust, the partners should decide which method of power determination fits HathiTrust's voting scheme. If votes will be cast simultaneously rather than sequentially, then the Banzhaf Index makes more sense than the Shapley-Shubik Index.

C. Determining Weight for a Given Banzhaf Power Allocation

Calculating the weights for a given power allocation requires a computer and a program to run the necessary algorithm. In Leech's 2003 paper, "Power Indices as an Aid to Institutional Design: The Generalised Apportionment Problem," he explains a step-by-step approach to calculating voting weights based on a given Banzhaf Index of power values. Essentially, his method relies on trial and error, starting with an arbitrary guess of weights and then repeating the guesses at certain intervals until hitting on the closest possible match. Keep in mind that, as Leech explains, there is no one unique weight distribution for every unique power index. Using his algorithm, it is possible to find an extremely close approximation, with a few important limitations: In general, the more voters there are, the more possible power distributions there are, and the easier it is to find weights to match a given power allocation. For example, Leech demonstrates that with 3 voters, for *any* combination of weights, there are *only four* possible power allocations. Up to 5 voters, there are still relatively few resulting power allocations.

Luckily for HathiTrust, by the time of the CC there will likely be at least seven partners, and the weights can be calculated to closely approximate most power allocations. In the spreadsheet attached to this document, each example HathiTrust power allocation corresponds to a certain weight distribution, and these weights are based on Leech's method. Credit for this work is due to Daniel Kneezel, a University of Michigan PhD student in mathematics, who designed a program to run Leech's algorithm. (For a more detailed explanation of this spreadsheet and specific recommendations for HathiTrust, see the last section of this document.)

III. Principles to Guide Voting Power Allocation

A priori voting power theory serves an extremely useful purpose: it enables the calculation of voting power based on unequal voting weights. It does not, however, make the job of deciding how to allocate voting power any easier. Many scholars, whether they incorporate formal voting power theory or not, have tackled the problem of how to allocate voting power in organizations with diverse constituents. Several general principles can be gleaned from these studies, as basic criteria to guide the distribution of voting power.

Barrett and Newcombe (1968) completed an extensive study of various weighted voting formulas and how they might apply to the United Nations. The authors neatly lay out four principles that a voting model should satisfy: "Decision-making by voting is a social invention designed to satisfy several types of demands: Those of justice (or equity), those of wisdom or effectiveness, those of reflecting and formalizing actual power relationships, and those of acceptability to the participants." (2) Corresponding to these "types of demands" are several reasons that weighted voting, rather than equal voting, is necessary in certain situations. The following table of principles ("types of demands"), with corresponding reasons for weighted voting, is adapted from Barrett and Newcombe's explanation on p. 1-5.

Types of Demands	Reasons for Weighted Voting
Equity	• Some may have a greater financial stake • Some may be more affected by the decisions • Some may have greater seniority rights • Some may represent larger organizations
Effectiveness	• Some may be in a better position to carry out the decisions • Some may be better informed about the issues
Reflecting and formalizing **power** relationships	• Some may have more personal power than others, and we wish to formalize this power rather than having it exercised informally by influencing the votes of others
Acceptability to participants	• No explicit reasons; weighted voting is "acceptable" when it reaches a compromise between equity and power

Looking at these demands and reasons, it is easy to see that many of them apply to the HathiTrust partnership. The specific permutations of the factors as they relate to HathiTrust will be discussed in the Recommendations section of this document. It is important to first understand the principles themselves, the reasons for those principles, and how real organizations attempt to satisfy those principles through weighted voting schemes.

A. Equity

The principle of equity is intuitive. In a situation where every voter has an equal say, the means of satisfying the principle of equity is simple: each person should have one vote. Many theorists argue that in real international organizations, whose members represent the citizens of their respective countries, the ideal voting power allocation will equalize the power of those citizens through weighted voting in the organization. If each member of the representative board gets one vote, then the citizens of larger countries actually have less power than the citizens of smaller countries. Likewise, Banzhaf and Shapley-Shubik have proven that giving larger countries a greater voting weight does not result in the appropriate allocations either. It was this principle of fairness of representation, and the concern that entire groups of citizens might not have any actual voting power, that drove Banzhaf to create the Banzhaf Index. A country's population, therefore, is often suggested as a main factor in determining voting power.

HathiTrust is not a representative body, and cannot rely on the democratic principle of one-person-one-vote. There are other factors, though, that are being "represented" by the voting members of

HathiTrust. These could include the number of volumes contributed to the repository. The greater number of volumes an institution has contributed, the greater the institution's stake in HathiTrust and the greater the amount of material that is being represented from that institution's collection.

B. Effectiveness

The principle of voter effectiveness is perhaps the most difficult to quantify, and yet it seems relevant to any organization in which members have varying degrees of experience. In HathiTrust, certain member institutions have more intimate familiarity with the governance of HathiTrust simply by having worked on the project at a granular level on a daily basis. If any new, non-founding partners join HathiTrust before the CC, certainly those new partners have less expertise than the current partners, as well as less capability to effectively act on decision making. Barrett and Newcombe classify the ability to act on decisions as part of the effectiveness principle. Voters who are wise and experienced are probably more involved in the issues at hand, and therefore better poised to carry out decisions. This correlation is certainly true in HathiTrust, where the members who have more experience also have more existing ties to the project and greater ability to carry out decisions.

C. Power

In any organization, some members have more real power than others. Barrett and Newcombe explain that there are several types of informal power a member might possess. For example, certain members may be more charismatic than others. Certain members may have more political power. The fact is that certain institutions in HT have more real power than others. Along with a commitment to host the physical infrastructure, Michigan and Indiana have made deeper commitments (e.g. the Michigan legal orientation, a larger financial commitment by each university, and IU's technological investments) that partners would be challenged to find at another institution.

The need to prevent stronger members from dominating decision-making, while still ensuring that smaller powers have a voice, is a central need that weighted voting attempts to solve. In their 2006 article "Reforming the IMF's Weighted Voting System," Rapkin and Strand (2006) argue that reforms to the IMF's voting regime must answer a "fundamental problem that is both theoretical and practical: how best to reconcile the principle of sovereign equality with the fact of wide power asymmetries among members." (p. 2) In the IMF, various members do have more real power than others, and there are certain voting measures that attempt to formalize that power. The United States, for certain types of decisions, "retains the only single-country veto over major IMF decisions, including any decision that would reduce its voting power and increase or decrease that of other countries." (p. 12) Veto power is one option in cases where one member simply has more real power than all of the other members put together. It can be harmful, however; Rapkin and Strand also point out that the U.S.'s veto power fosters "perceptions of systemic unfairness" (p. 2) and influences decisions even when it is not explicitly exercised.

Whether an organization grants certain members veto power or not, the fact remains that more powerful members of an organization will not accept a voting model in which they are not granted enough power. Neither will the less powerful members have confidence in a model that grants them very little or no power. The principle of power, therefore, is highly interrelated with the principle of acceptability.

D. Acceptability

Acceptability, according to Barrett and Newcombe (among others), represents a compromise between the principles of power and equity. An acceptable scheme will represent the interests of both the stronger and weaker parties. Acceptability is arguably the most important principle that a weighted voting model should satisfy, simply because a voting model that is not acceptable to voters will not work. Newcombe, Wert, and Newcombe (1971) also echo this focus on acceptability in their later study of possible UN formulas:

> Only a world body that has the confidence of as many member-nations as possible, East and West, rich and poor, large and small, will be able to obtain the powers it needs. The 'balancing' needed to gain the confidence of member states is a difficult matter. Precisely this balancing is the purpose of any weighted-voting scheme. . . . The main criterion in evaluating a weighted-voting formula is *acceptability*, not either abstract justice or theoretical reasoning." (452)

Their astute observation is a good one to keep in mind, and will most likely be the driving principle behind selecting a voting procedure for any organization. In the long term, no voting procedure will work if it does not inspire the confidence of the constituents.

Many of the vote allocation formulas proposed by Barrett and Newcombe, and later discussed by Dixon (1983), in relation to the United Nations, aim at some sort of compromise between the factors of equity and power. The factor of a country's population correlates to the demands of equity, while the factor of GDP corresponds to "a rough indication of a state's power." (Dixon, 1983) These functions include taking direct numbers, logarithms, square roots, and other variations on each country's respective GDP and population. Interestingly, Dixon also debates the acceptability of *a priori* voting theory itself, as the point of his study is to apply the Banzhaf Index to each formula. He points out that if a weighted voting system is designed specifically to meet the needs of a given power allocation, the voting weights necessary to generate a given power distribution will seem so arbitrary to voters as to render them unacceptable.

One possible compromise between weighted voting and equal voting, which is Rapkin and Strand's (2006) most important recommendation to the IMF, is the system of voting by double majority. The "Count and Account" (double majority) voting scheme, as described below by O'Neill and Peleg (2000), represents a more stable and consistent compromise between weighted voting and equal representation:

> Voting by count and account takes into consideration both size and equality and does so in a simple way. Votes are counted twice, first with each party weighted equally, and then with each weighted by its financial contribution or some other objective measure. A proposal passes if it gets a majority in both ways. The rule formalizes the idea that an organization should act only when it has the support of both the general membership and the important members. (3)

One thing to be aware of is that the count and account scheme is "necessarily more conservative than using a straight majority of the account weights alone, since two criteria have to be satisfied rather than one and fewer resolutions get passed. This can be seen either as a disadvantage of the method." (O'Neill and Peleg 8) It is also possible to apply voting power indexes to the double majority method; Turnovec (1997) has done so in his useful study of voting in the European Union. The double majority method is one possible way to increase acceptability. It could be particularly useful as a technique for HathiTrust.

IV. Recommendations for a HathiTrust Voting Model

Drawing from the previous sections' discussions of *a priori* voting power theory as well as case studies of voting power allocation, this section will propose recommended voting models for HathiTrust. Recommendations fall into two different categories: basic recommendations, and specific recommendations. Whereas basic recommendations have to do with the assumptions and scaffolding of the voting model, specific recommendations will entail a number of different formulas that could work for HathiTrust.

To design a voting model that works for HathiTrust, we must ask:

- Which of the four principles—equity, effectiveness, power, and acceptability—are relevant to HathiTrust at the Constitutional Convention?
- How can the principles be satisfied through a weighted voting model, and what factors are available to inform the model?

To answer the first question, it is helpful to turn to existing HathiTrust documentation, particularly the Mission and Goals, FAQ, and Functional Objectives on the HathiTrust website (http://www.hathitrust.org/). Across all of the documentation, there is a strong focus on collaboration, co-ownership, and openness. The Mission and Goals explain HathiTrust's core values, stating that the organization is a "collaboration of the thirteen universities of the Committee on Institutional Cooperation and the University of California system to establish a repository for these universities to archive and share their digitized collections. Partnership is open to all who share this grand vision." At the same time, the HathiTrust documentation makes references to the fact that despite this unity of vision, partners will retain their individuality. For example, HathiTrust will be "co-owned and managed by a number of academic institutions" and will strive to "dramatically improve access to these materials in ways that, first and foremost, meet the needs of the co-owning institutions." These statements suggest at once collective ownership, collective management, and collective needs, but also individual ownership, individual management, and individual needs. It is safe to say that HathiTrust is a collaborative organization with a cohesive vision that is nonetheless comprised of individual and distinct constituents. One of the challenges in designing a voting model will be recognizing the fundamental value of each HathiTrust institution, while allowing for differences in power and influence.

It is also clear from the HathiTrust documentation that the founding partners of HathiTrust deserve a great deal of credit for their hard work on the project thus far. Under the FAQ "Who's taking the lead?"

the following statement emphasizes the commitment and expertise of the current partners: "The University of Michigan, Indiana University, the University of Virginia, and the University of California system, all highly regarded for their expertise in the areas of information technology, digital libraries, and project management, are leading the partnership effort through their expertise and financial commitment. All members of the CIC are founding partners." Recognizing power differences among the partners suggests that the principles of both equity and power need to be satisfied by a weighted voting system. Currently, there are five partners:

- University of Michigan
- Indiana University
- California Digital Library (includes all the University of California institutions)
- Committee on Institutional Cooperation (excluding Michigan and Indiana)
  - Michigan State University
  - Northwestern University
  - The Ohio State University
  - Penn State University
  - Purdue University
  - The University of Chicago
  - University of Illinois
  - University of Illinois at Chicago
  - The University of Iowa
  - University of Minnesota
  - University of Wisconsin-Madison
- University of Virginia

In addition, Columbia University and Yale University may also be joining the ranks of HathiTrust relatively soon. Among the partners, the following factors contribute to their differences in commitment to HathiTrust:

- Financial contributions
- Projected number of volumes in repository
- Length of commitment (since January 2008)
- Repository administrator status
  - University of Michigan, Indiana University
- Founder status
  - University of Michigan, Indiana University, Committee on Institutional Cooperation, University of California

To illustrate the relationship of these factors to principles, or "types of demands," the following table mimics the previous table while adding in each factor and where it would correlate to a principle:

Types of Demands	HathiTrust Reasons for Weighted Voting	HathiTrust Weighting Factors
Equity	• Some have a greater financial stake • Some are more affected by the decisions due to	• Financial contribution • Number of volumes contributed (projected) • Length of commitment,

Equity	• Some have a greater financial stake • Some are more affected by the decisions due to greater investment of resources and contribution of volumes • Some have made a longer commitment and/or or are founding members	• Financial contribution • Number of volumes contributed (projected) • Length of commitment, starting in January 2008 • Founder status
Effectiveness of Voters	• Some are HathiTrust administrators • Some have been more involved than others in shaping HathiTrust, and are more informed about the issues	• Administrator status • Length of commitment, starting in January 2008
Power	• Some are HathiTrust administrators and/or founding members, and therefore have more real	• Administrator status • Founder status

In this table, several of the factors overlap; that is, they fulfill several different principles. The principle of effectiveness is the only one that does not have any unique factors. It is safe to say that equity, power, and acceptability are the demands that will determine the HathiTrust CC voting model. Effectiveness is certainly important, but any voting model that satisfies the other three principles will automatically satisfy the principle of effectiveness as well.

A. General Recommendations

The recommendations for HathiTrust's voting model can be divided into general recommendations, pertaining to the overarching structure of the model, and specific recommendations pertaining to the more complicated subset of questions about power allocation and voting weight. Those more complicated questions will be discussed in Section B. The following set of recommendations applies to the basic architecture of a HathiTrust voting power scheme. It goes without saying that they are meant to satisfy the principles of equity, power, and acceptability, as well as appropriately reflect the factors .

- Implement a double majority voting system of both weighted and equal votes, requiring a majority of both types of votes in order for a decision to pass
- For the weighted votes, set a simple majority (50% quota)
- For the equal votes, the majority can be set at a simple majority or higher depending on the importance of the decision
- For especially important decisions, it would also be wise to require a majority or unanimity of the HathiTrust founding institutions: UM, UC, CIC, and IU

- HathiTrust administrators, UM and IU, should each have an institutional veto
- Stick to binary (two options, or yes/no) voting, which will enable the application of *a priori* voting power theory to the weighted votes
- Determine a voting power allocation based on an appropriate formula and factors, and then calculate the voter weights according to the Banzhaf Index using Leech's (2003) method (see Section B and attached spreadsheet)
    - Recommended power allocation formula:

      Voting Power = (square root of volumes) + (square root of financial contribution)

Most of these recommendations are self-explanatory, or have been justified elsewhere in the document already. The double majority voting model effectively reflects the tension evident in the HathiTrust mission statement between institutional equality and power differences. It fulfills the principle of acceptability because it achieves in execution what it promises in theory. It is also an extremely useful model, as it is simple and yet adaptable to different types of decision-making. This adaptability makes it likely to be an acceptable scheme. As noted above, the equal votes portion of the scheme can be easily manipulated in terms of its quota and also the subsets of types of voters. For important decisions—moving the HathiTrust host institution, for example—it makes sense to require not only a majority of voters, but a higher majority than usual and perhaps a majority of founding member institutions.

Implementing an institutional veto for the HathiTrust administrators is likely to be the most controversial of these recommendations, but it makes sense both practically and theoretically. The University of Michigan and Indiana University have both made tremendous commitments of administrative resources and legal responsibility to HathiTrust.

As for the weighted voting portion of the scheme, the quota is less flexible. The quota should be set at 50% because, with a higher quota, it is simply more difficult to find weights that accurately reflect a given power allocation (see Leech 2003). Since the goal of weighted voting is to achieve an accurate allocation of power, it makes sense to use a quota that achieves this accuracy. The adherence to binary votes is also meant to satisfy the need for accurate voting weights, as only binary voting can be analyzed using *a priori* methods.

The recommendation of using the Banzhaf Index, as opposed to the Shapley-Shubik Index of calculating voting power, is based on the two methods' very different conceptions of voter motivation and procedure. The Banzhaf Index assumes that voters are disinterested parties (policy-seeking) rather than seekers of their own advancement. The HathiTrust partners certainly have the project's best interests at heart, and therefore the Banzhaf conception of motivation is more appropriate. The Shapley-Shubik Index is also based on sequence of votes, which seems irrelevant to HathiTrust. Votes at the HathiTrust CC should be taken simultaneously. As for the voting weights themselves, there are a number of different formulas that can be used to allocate power, but some work better to accurately reflect institutional contributions and achieve a balance of power and equity.

B. Specific Recommendations: Power Allocation and Voting Weights

The following recommendations refer to the first point of the basic recommendations: "Determine a voting power allocation based on an appropriate formula and factors, and then calculate the voter weights according to the Banzhaf Index using Leech's (2003) method." The attached spreadsheet illustrates several different power allocations according to formulas, as well as the weights determined by Leech's algorithm discussed above. As founder status and administrator status have already been accounted for through other recommendations, these formulas are derived from the quantifiable factors of financial contributions and number of volumes contributed to HathiTrust.

There are several possible power allocations that could work, and this section will propose four formulas. As mentioned in the previous section, Dixon (1983), drawing from Newcombe, Wert, and Newcombe (1971) as well as Barrett and Newcombe (1968), proposed several formulas that purport to satisfy the principles of equity and power (and thus achieve acceptability) by taking either direct proportions or simple mathematical derivations of both population and GDP. Recall that population satisfies the principle of equity, while GDP equates to power.

The formulas are as follows:

1. Population + GDP
2. Population x square root of GDP per capita
3. Log population + log GDP

The first formula obviously directly correlates to the two factors involved, simply totaling the two. The third formula uses "the application of logarithms as a way to discount extreme values," (p. 300) while the second "compensates for population differences by distributing votes according to the product of population and the square root of GDP per capita." (p. 300) Out of Dixon's fifteen formulas, these are the three that use multiple factors and also use mathematical functions to specifically mitigate the inequalities of the raw numbers. They seem appropriate and translatable to HathiTrust's needs. (Note that these formulas were originally proposed as formulas for *weight*, but what they are truly meant to signify is *power*. It is safe to use them to allocate power rather than weight.)

While HathiTrust members do not have either population or GDP, there are certain factors in HathiTrust that satisfy the principles of both equity and power, respectively, and could be substituted in the formulas. The principle of equity, for HathiTrust, could be roughly fulfilled by a factor of the number of volumes contributed to the repository. The principle of power, meanwhile, could be signified by financial contribution. Substituting these factors for the original ones, we have:

1. Number of volumes + financial contribution
2. Number of volumes x square root of financial contribution
3. Log (number of volumes) + log (financial contribution)

To these formulas can be added a fourth formula that does not appear in the literature but reflects some of the reasoning behind the 2nd and 3rd formula above, and turns out to be better for HathiTrust's needs:

4. Square root of volumes + square root of financial contribution

This formula will be the recommended formula for HathiTrust, but let us explore how the four formulas compare.

First, here are the projected estimated values for each HathiTrust institution's financial contribution and volumes contributed, as of March 2011. These numbers are merely guesses at this point, and can be adjusted in the future:

Institution	**Financial contribution**	**Volumes**
University of Michigan	2,155,404	5,000,000
Indiana University[1]	900,000	600,000
Committee on Institutional Cooperation	1,650,000	1,000,000
University of California	1,244,142	3,000,000
University of Virginia	72,000	500,000
Columbia University	72,000	500,000
Yale University	40,000	30,000
TOTALS	6133546	10,630,000

It is more equitable to use these projected amounts, rather than current amounts. Some institutions have more volumes currently in the repository simply because they were there first. In addition, as explained previously, Leech's algorithm of calculating voting weight for a given power distribution works far better on groups of more than five voters.

The attached spreadsheet shows different power and weight values of these four formulas, according to the institution. The *power* values were calculated according to the formulas, while the *weights* were calculated separately using Leech's algorithm. Note that changing the input values of volumes or financial contributions will readjust the power values, but will not re-calculate the weights. Looking at the values that result from each of these formulas, it is immediately apparent that certain formulas compensate for numerical extremes better than others, and that such compensation is an extremely important criteria for selecting a power allocation formula.

In formula 1, the University of Michigan has 43% of the total power, while Yale University receives only 0.42% of the power. Such an extreme difference would most likely be considered unacceptable and unfair by all of the partners, and certainly by Yale. The following table gives the formula 1 power and weight values:

Formula 1: **(financial contribution) + (volumes)**

[1] To generate IU's voting power, IU's volume estimates have been separated from the total CIC volumes.

Institution	Power	Relative Power[2]	Relative Weights[3]
University of Michigan	7,155,404	42.68%	41.73%
Indiana University	1,500,000	8.95%	7.81%
Committee on Institutional Cooperation	2,650,000	15.81%	15.60%
University of California	4,244,142	25.32%	26.73%
University of Virginia	572,000	3.41%	3.87%
Columbia University	572,000	3.41%	3.87%
Yale University	70,000	0.42%	0.37%
TOTALS	16,763,546	100%	100%

One positive aspect of formula 1 is that the power ratios are very close to the weight ratios. This strong correlation makes the weights more intuitive and therefore more acceptable. Still, the wildly uneven power allocation makes this formula less than ideal.

Formula 2 results in even greater highs and lows of power values, and some of the weights generated as a result of these extremes are zero or negative values. Obviously, negative weight values are useless from a practical and theoretical standpoint. Here are the values resulting from formula 2:

Formula 2: volumes x square root of (financial contribution)			
Institution	**Power**	**Relative Power**	**Relative Weights**
University of Michigan	7,340,647,110	57.28%	50.08%
Indiana University	569,209,979	4.44%	0.00%
Committee on Institutional Cooperation	1,284,523,258	10.02%	0.06%
University of California	3,346,233,405	26.11%	50.01%
University of Virginia	134,164,079	1.05%	-0.07%
Columbia University	134,164,079	1.05%	-0.07%
Yale University	6,000,000	0.05%	-0.01%
TOTALS	12,814,941,909	100%	100.00%

Note, also, that the power values and weight values according to this formula are quite different. A relative power of 10.02% yields a weight of 0.06%, which seems counterintuitive even if it is mathematically correct.

Unlike Formulas 1 and 2, which result in extreme highs and low of power allocations, Formula 3 attempts to alleviate these extremes by taking the logarithm of each factor. The resulting power allocation is perhaps not diverse enough, with all of the percentages lying very close together:

[2] Relative Power represents the institution's power divided by the total power of all institutions.
[3] Relative Weights represent the closest possible match of weight ratios, according to Leech's algorithm, that yield the desired power allocation. Any weights that maintain the given ratios will result in the same power allocation. The simplest way to maintain the ratios would be to allocate votes as integers totaling 100.

Formula 3: log (financial contribution) + log (volumes)			
Institution	**Power**	**Relative Power**	**Relative Weights**
University of Michigan	13.03	16.34%	17.51%
Indiana University	11.73	14.71%	14.96%
Committee on Institutional Cooperation	12.22	15.32%	15.91%
University of California	12.57	15.77%	16.61%
University of Virginia	10.56	13.24%	12.64%
Columbia University	10.56	13.24%	12.64%
Yale University	9.08	11.39%	9.73%
TOTALS	79.75	100.00%	100%

So far, two of the three formulas have resulted in too many extreme values, and the third yields a power allocation that is *too* even given the variation in partner commitments to HathiTrust. Evidently, as with the overall voting model design, it is difficult with these formulas to achieve a middle ground between extreme inequality and complete equality. The 4th formula, derived in response to this need for a middle ground, seems to achieve the necessary balance. Those values are as follows:

Formula 4: square root of (financial contribution) + square root of (volumes)			
Institution	**Power**	**Relative Power**	**Relative Weights**
University of Michigan	3,704	28.75%	28.30%
Indiana University	1,723	13.38%	13.98%
Committee on Institutional Cooperation	2,285	17.73%	17.84%
University of California	2,847	22.10%	21.69%
University of Virginia	975	7.57%	7.70%
Columbia University	975	7.57%	7.70%
Yale University	373	2.90%	2.78%
TOTALS	12,884	100%	100%

Taking the square root of each factor, and then totaling them, mitigates the problem of extremes, but not as thoroughly as taking the logarithm. It seems to represent an acceptable distribution of power, and in addition, the corresponding weights are extremely close to the actual power distribution. The closeness of weight and power serves to make this formula more acceptable and intuitive. The final recommendation for HathiTrust, therefore, is to use formula #4 to allocate power, equating voting power with the sum of square roots of an institution's financial contribution and number of volumes contributed. This weighted voting formula will serve as a core component of a larger double majority scheme, satisfying the principles of both equality and power disparities in HathiTrust. To see how this proposed voting model would play out in real decision-making, let us examine a few potential scenarios.

C. Potential Voting Scenarios

Although we can only guess at configurations of voter preferences for any given question, it is worthwhile to look at how the votes would play out for a few hypothetical situations. At the CC, the partners will respond to recommendations from the HathiTrust three-year review. Imagine that one of these recommendations pertains to whether or not the HathiTrust model should continue to combine preservation and access. The 3-year review recommends that HathiTrust continue to embrace the concept of a "light archive," enabling both preservation and access. Now imagine that of the partners except for the University of Michigan support the recommendation. The University of Michigan opposes this measure and supports converting HathiTrust into a "dark archive" on the grounds that it is more economical and less complex. According to formula 4, the weighted votes would play out as follows:

Institution	**Weights (Integers)**	**Maintain "light archive"?**
University of Michigan	28.3	NO
Indiana University	14.0	YES
Committee on Institutional Cooperation	17.8	YES
University of California	21.7	YES
University of Virginia	7.7	YES
Columbia University	7.7	YES
Yale University	2.8	YES
TOTALS/OUTCOME	100.0	YES
NUMBER OF YES	n/a	71.7
NUMBER OF NO	n/a	28.3

To make the table more readable, the weights have been turned into positive numbers totaling 100, rounded off to the nearest tenth. With all of the partners except for University of Michigan coming in at 71.7% of the vote, their preference to keep the light archive model will pass in the weighted voting phase.

According to the other components of the recommended voting model, there is also the double majority requirement to be considered, as well as the fact that Michigan has a veto power. This scenario also easily passes according to the unweighted votes component of the double majority, with six out of seven members in agreement. It also happens, though, that the one member in disagreement has veto power. The fact that the University of Michigan possess the ability to veto does not automatically mean that UM would exercise that power. In this hypothetical situation, Michigan would have to carefully consider the opinions of the other partners as well as the political ramifications of opposing the collective membership. If Michigan did choose to veto the decision, it would not automatically mean that the vote would go Michigan's way; rather, the question would require further discussion, possible alterations to the proposal, and another vote.

Taking this same hypothetical proposition of maintaining HathiTrust as a "light archive," now suppose that Indiana and California join Michigan in opposing the measure and voting to convert HathiTrust into a dark archive. The weighted votes would look like this:

Institution	**Weights (Integers)**	**Maintain "light archive"?**
University of Michigan	28.3	NO
Indiana University	14.0	NO
Committee on Institutional Cooperation	17.8	YES
University of California	21.7	NO
University of Virginia	7.7	YES
Columbia University	7.7	YES
Yale University	2.8	YES
TOTALS/OUTCOME	100.0	NO
NUMBER OF YES	n/a	36.0
NUMBER OF NO	n/a	64.0

In this scenario, the "no's" get the weighted vote with a 64% majority. But taking into consideration the double majority rule, the "yes" votes have four out the seven unweighted votes. The measure neither passes nor fails, and the partners go back to discussion. In a situation like this one, where there are conflicting outcomes between the weighted and unweighted votes, it is clear that having a double majority rule makes passing a decision more difficult. This could be seen in a positive light—an opportunity for the partners to have further discussion and perhaps modify the proposal—or in a negative light, as an obstruction to decision-making.

There are an infinite number of scenarios to be considered, and to this purpose, the attached spreadsheet includes a "vote calculator" based on formula 4.

V. Conclusion

To revisit the initial goals put forth in this document, the questions that this document set out to answer were:

- How is voting power calculated in a weighted voting system?
- What factors and principles should inform allocation of voting power?
- How do these models apply to HathiTrust?

These three questions were answered through an exploration of *a priori* voting power theory, an examination of the basic principles that various theorists have suggested, and finally, a detailed set of recommendations for HathiTrust along with several potential scenarios and their outcomes according to the recommended voting model. That recommended model, which involves three components—weighted votes, unweighted votes, and institutional veto power—attempts to satisfy the principles of equity, power, and acceptability detailed by theorists and also evident in HathiTrust's online documentation of the project's mission.

The goal of this document has been not only to recommend a specific voting model for HathiTrust as it is comprised in the foreseeable future, but also to outline flexible approaches and formulas that can be adapted as the membership and contributing factors (financial contribution and number of volumes) change. For example, if the host institution changes, it may no longer be practical to give the University of Michigan veto power. In the weighted voting component, it might make more sense to use a different formula for power allocation if the factors change; formula 3, for instance, equalizes power too much in the current configuration. With more extreme differences in contributing factors (financial contributions and number of volumes), formula 3 might work to offset those differences. In addition, the executive committee may wish to consider different breakdowns of the membership, such as breaking the CIC and UC into their respective member institutions to be unique HathiTrust partners. As the HathiTrust Digital Library will certainly evolve in unexpected ways, its decision-making processes should also evolve to support those changes.

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